## Statistics Explanation of Population Proportion Confidence Interval (CI) Formula

**Example**: What proportion of ESU students drink coffee? We sampled 155 ESU students, and 82 of them told us they drink coffee.

Population = all ESU students	Sample = 155 ESU students who were asked $(n = 155)$
Parameter = $p$ = proportion of all ESU students	Statistic = $\hat{p}$ = proportion of the sample who drink coffee
who drink coffee (unknown)	

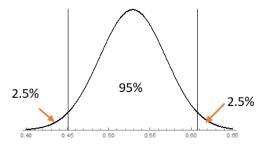
$$\hat{p} = \frac{82}{155} = 0.529$$

<u>From Chapter 6</u>: The Distribution of  $\hat{p}$  is approximately normal

	Actual	Approximated
Mean	p	$\hat{p} = 0.529$
SD	$\sqrt{\frac{p(1-p)}{n}}$	SE = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.529(1-0.529)}{155}} = 0.04$

<u>Idea Behind the CI</u>: The unknown parameter p is in the center of the distribution. By using the unbiased estimator  $\hat{p}$  as the center and approximating the middle 95% of the distribution, we have a 95% chance that we found p.

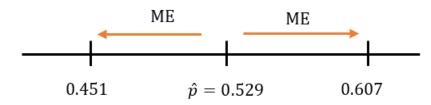
We get a 95% Confidence Interval for p by finding the middle 95% of this distribution



invNorm(0.025, 0.529, 0.04) = 0.451 invNorm(0.975, 0.529, 0.04) = 0.607

- A 95% CI for the proportion of ESU students who drink coffee is (0.451, 0.607)
- <u>Correct Interpretation</u>: There's a 95% chance that the interval (0.451, 0.607) contains the proportion of ESU students who drink coffee.

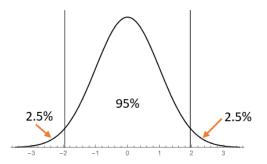
Picture of the Confidence Interval



- The Margin of Error (ME) is 0.607 0.529 = 0.078 (you can find this in a few different ways)
- For the CI, we start at  $\hat{p}$  in the center and then add and subtract ME.
- Another way to write the CI is

$$\hat{p} \pm ME = 0.529 \pm 0.078$$

The Margin of Error formula is based on the Standard Normal Curve *z* (Mean 0, SD 1). Here is the middle 95%:



invNorm(0.025, 0, 1) = -1.96

invNorm(0.975, 0, 1) = 1.96

The *z*-score 1.96 is called a <u>critical value</u>

In the example, we had ME = 0.078, SE = 0.04

Notice that ME = 0.078 = 1.96 \* 0.04 = 1.96 \* SE

## Formula for the Margin of Error

$$ME = z^*(SE) = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

We put it all together to get:

## Formula for the Confidence Interval for p

Point Estimate 
$$\pm$$
 ME =  $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$