## Statistics <br> Explanation of Population Proportion Confidence Interval (CI) Formula

Example: What proportion of ESU students drink coffee? We sampled 155 ESU students, and 82 of them told us they drink coffee.

| Population $=$ all ESU students | Sample $=155$ ESU students who were asked $(n=155)$ |
| :--- | :--- |
| Parameter $=p=$ proportion of all ESU students <br> who drink coffee (unknown) | Statistic $=\hat{p}=$ proportion of the sample who drink coffee |

$$
\hat{p}=\frac{82}{155}=0.529
$$

From Chapter 6: The Distribution of $\hat{p}$ is approximately normal

|  | Actual | Approximated |
| :--- | :---: | :---: |
| Mean | $p$ | $\hat{p}=0.529$ |
| SD | $\sqrt{\frac{p(1-p)}{n}}$ | $\mathrm{SE}=\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=\sqrt{\frac{0.529(1-0.529)}{155}}=0.04$ |

Idea Behind the CI: The unknown parameter $p$ is in the center of the distribution. By using the unbiased estimator $\hat{p}$ as the center and approximating the middle $95 \%$ of the distribution, we have a $95 \%$ chance that we found $p$.

We get a 95\% Confidence Interval for $p$ by finding the middle $95 \%$ of this distribution

$\operatorname{invNorm}(0.025,0.529,0.04)=0.451$
$\operatorname{invNorm}(0.975,0.529,0.04)=0.607$

- A 95\% CI for the proportion of ESU students who drink coffee is $(0.451,0.607)$
- Correct Interpretation: There's a $95 \%$ chance that the interval ( $0.451,0.607$ ) contains the proportion of ESU students who drink coffee.

Picture of the Confidence Interval


- The Margin of Error (ME) is $0.607-0.529=0.078$ (you can find this in a few different ways)
- For the CI, we start at $\hat{p}$ in the center and then add and subtract ME.
- Another way to write the CI is

$$
\hat{p} \pm M E=0.529 \pm 0.078
$$

The Margin of Error formula is based on the Standard Normal Curve z (Mean 0, SD 1). Here is the middle 95\%:

$\operatorname{invNorm}(0.025,0,1)=-1.96$
$\operatorname{invNorm}(0.975,0,1)=1.96$
The $z$-score 1.96 is called a critical value

In the example, we had $\mathrm{ME}=0.078, \mathrm{SE}=0.04$
Notice that ME $=0.078=1.96$ * $0.04=1.96$ *SE

## Formula for the Margin of Error

$$
M E=z^{*}(S E)=z^{*} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
$$

We put it all together to get:

Formula for the Confidence Interval for $p$
Point Estimate $\pm \mathrm{ME}=\hat{p} \pm z^{*} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

